

**APPROVED**

By COHONGTRAN at 9:58 am, Aug 31, 2010



## Chương I. HÀM SỐ LƯỢNG GIÁC - PHƯƠNG TRÌNH LƯỢNG GIÁC

### § 2. Phương trình lượng giác :

#### 1. Phương trình cơ bản :

$$\sin u = \sin v \Leftrightarrow \begin{cases} u = v + k2p \\ u = p - v + k2p \end{cases} \quad k \in Z$$
$$\cos u = \cos v \Leftrightarrow \begin{cases} u = v + k2p \\ u = -v + k2p \end{cases} \quad k \in Z$$
$$\tan u = \tan v \Leftrightarrow u = v + kp, k \in Z \quad (u, v \neq \frac{p}{2} + hp)$$

Lưu ý :

\*  $\sin u$  ( hoặc  $\cos u$  ) =  $m$  có nghiệm khi  $-1 \leq m \leq 1$

\*

$$\sin u = 0 \Leftrightarrow u = kp$$

$$\sin u = \pm 1 \Leftrightarrow u = \pm \frac{p}{2} + k2p$$

$$\cos u = 0 \Leftrightarrow u = \frac{p}{2} + kp$$

a. Dạng cơ bản : giải bình thường .

Ví dụ :  $\sin\left(2x + \frac{\pi}{6}\right) = \sin\left(3x + \frac{\pi}{4}\right)$

> `ptcb(sin, 2*x+Pi/6, 3*x+Pi/4);`

" Cho phương trình : ",  $\sin\left(2x + \frac{\pi}{6}\right) = \sin\left(3x + \frac{\pi}{4}\right)$

" Phương trình tương đương "

$$2x + \frac{\pi}{6} = 3x + \frac{1}{4}\pi + k2\pi$$

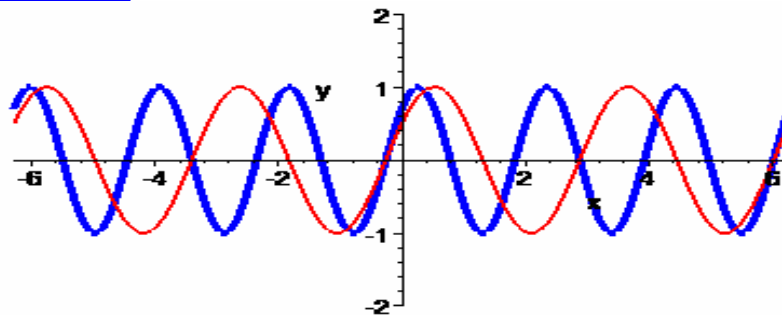
$$2x + \frac{\pi}{6} = \frac{3}{4}\pi - 3x + k2\pi$$

$$\{x = -\frac{1}{12}\pi - k2\pi\}$$

$$\{x = \frac{7}{60}\pi + \frac{1}{5}k2\pi\}$$

$k, \varepsilon, Z$

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— vp-ĐỎ  
 — vt-XANH

Ví dụ :  $\sin\left(2x + \frac{\pi}{3}\right) = -\frac{\sqrt{2}}{2}$

> `ptcb(sin,2*x+Pi/3,(-3*Pi/4));`

" Cho phương trình : ",  $\sin\left(2x + \frac{\pi}{3}\right) = -\frac{\sqrt{2}}{2}$

" Phương trình tương đương "

$$2x + \frac{\pi}{3} = -\frac{3}{4}\pi + k2\pi$$

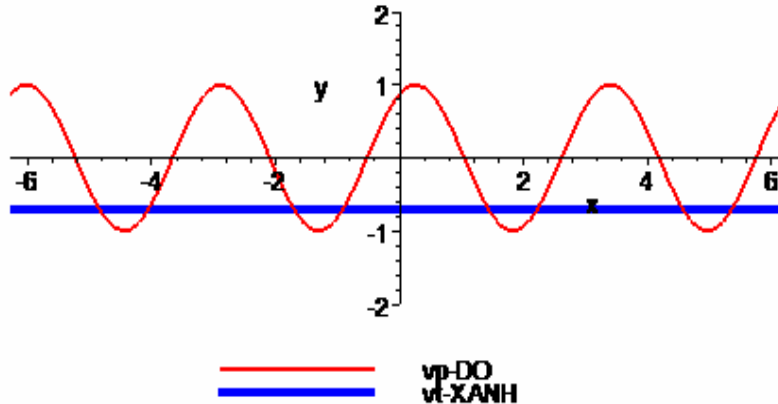
$$2x + \frac{\pi}{3} = \frac{7}{4}\pi + k2\pi$$

$$\{x = -\frac{13}{24}\pi + \frac{1}{2}k2\pi\}$$

$$\{x = \frac{17}{24}\pi + \frac{1}{2}k2\pi\}$$

$k, \varepsilon, Z$

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b. Dạng khác hàm : dùng cung PHỤ (  $\frac{p}{2} - (.)$  ).

$$\sin\left[\frac{p}{2} - (.)\right] = \cos(.); \quad \cos\left[\frac{p}{2} - (.)\right] = \sin(.)$$
$$\tan\left[\frac{p}{2} - (.)\right] = \cot(.); \quad \cot\left[\frac{p}{2} - (.)\right] = \tan(.)$$

Ví dụ :

$$\sin\left(x + \frac{\pi}{5}\right) = \cos\left(3x - \frac{\pi}{10}\right)$$

> `ptcbkh(sin,x+Pi/5,cos,3*x-Pi/10);`

" Cho phương trình : ",  $\sin\left(x + \frac{\pi}{5}\right) = \cos\left(3x - \frac{\pi}{10}\right)$

" Dùng cung phụ biến đổi hàm sin về hàm cos "

$$\cos\left(\frac{3\pi}{10} - x\right) = \cos\left(3x - \frac{\pi}{10}\right)$$

" Phương trình tương đương "

$$\frac{3\pi}{10} - x = 3x - \frac{1}{10}\pi + k2\pi$$

$$\frac{3\pi}{10} - x = -3x + \frac{1}{10}\pi + k2\pi$$

$$\left\{ x = \frac{1}{10} \pi - \frac{1}{4} k 2 \pi \right\}$$

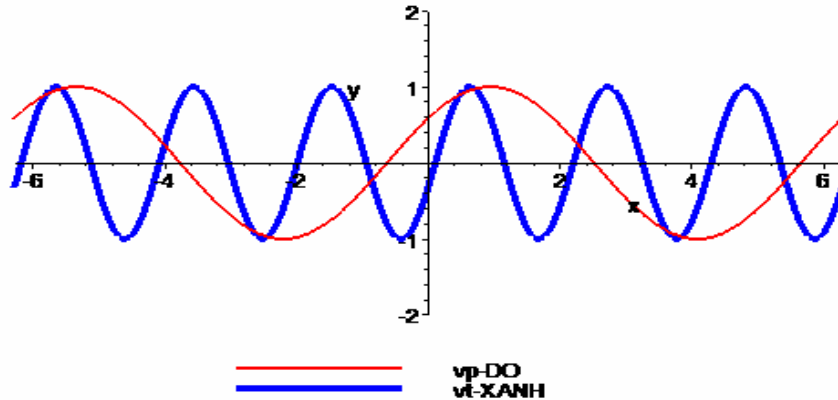
$$\left\{ x = -\frac{1}{10} \pi + \frac{1}{2} k 2 \pi \right\}$$

$$k, \varepsilon, Z$$

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>

$$\text{Ví dụ : } \tan\left(5x + \frac{\pi}{3}\right) = \cot\left(x + \frac{\pi}{6}\right)$$

> **ptcbkh(tan, 5\*x+Pi/3, cot, x+Pi/6);**

$$\text{" Cho phuong trinh : ", } \tan\left(5x + \frac{\pi}{3}\right) = \cot\left(x + \frac{\pi}{6}\right)$$

" Dung cung phu bien doi ham cot ve ham tan "

$$\tan\left(5x + \frac{\pi}{3}\right) = \tan\left(\frac{\pi}{3} - x\right)$$

$$5x + \frac{\pi}{3} = \frac{1}{3} \pi - x + k \pi$$

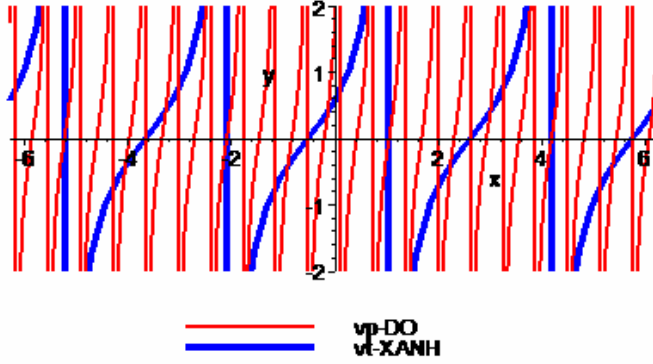
$$\left\{ x = \frac{k \pi}{6} \right\}$$

$$k, \varepsilon, Z$$

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c. Dạng có dấu (-) : đưa dấu trừ vào cung , dùng các công thức sau

$$\begin{aligned}
 -\sin(\quad) &= \sin(-\quad) \\
 -\tan(\quad) &= \tan(-\quad) \\
 -\cos(\quad) &= \cos(p - \quad)
 \end{aligned}$$

Ví dụ :  $\cos\left(2x + \frac{1}{3}\pi\right) = -\cos\left(3x + \frac{1}{4}\pi\right)$

> **ptcbdt(cos, 2\*x+Pi/3, 3\*x+Pi/4);**

" Cho phương trình : ",  $\cos\left(2x + \frac{1}{3}\pi\right) = -\cos\left(3x + \frac{1}{4}\pi\right)$

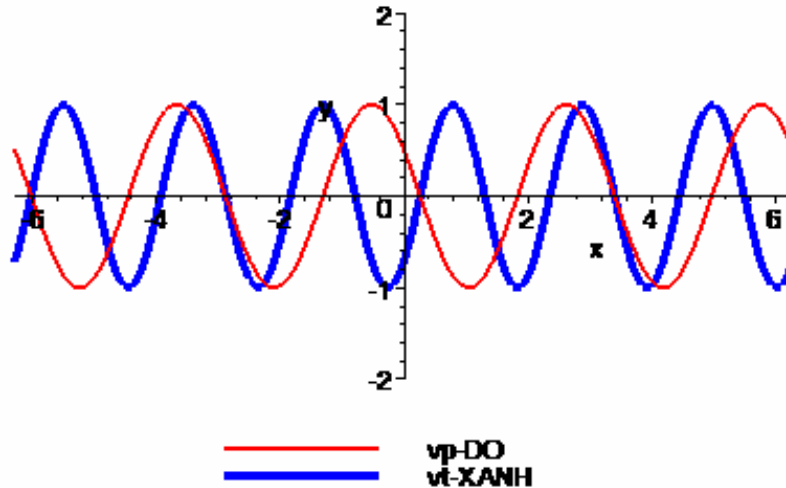
" Đưa dấu trừ vào cung :  $-\cos(\quad) = \cos(\pi - \quad)$  "

" Phương trình  $\Leftrightarrow$  ",  $\cos\left(2x + \frac{1}{3}\pi\right) = \cos\left(\frac{3}{4}\pi - 3x\right)$

$$\left\{ x = \frac{1}{12}\pi + \frac{1}{5}k2\pi \right\}$$

$$\left\{ x = \frac{13}{12}\pi - k2\pi \right\}$$

$$k, \varepsilon, Z$$



Ví dụ :  $\tan\left(5x + \frac{1}{4}\pi\right) = -\tan\left(x + \frac{1}{6}\pi\right)$

> `ptcbdt(tan,5*x+Pi/4,x+Pi/6);`

" Cho phương trình : ",  $\tan\left(5x + \frac{1}{4}\pi\right) = -\tan\left(x + \frac{1}{6}\pi\right)$

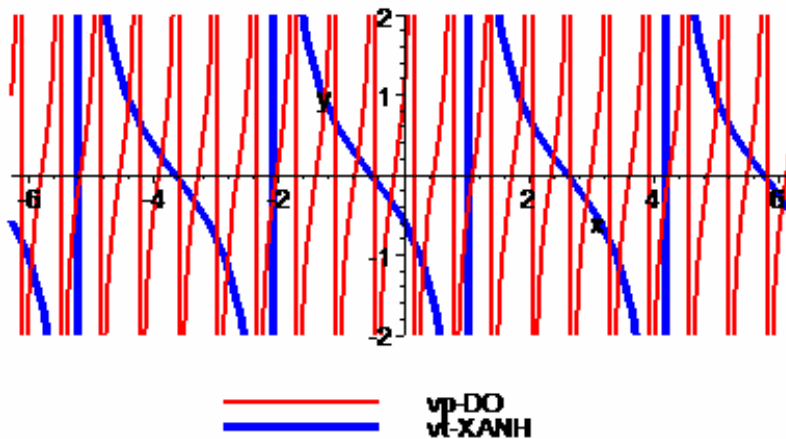
" Dưa dấu trừ vào cùng :  $-\tan(\ ) = \tan(-\ )$  "

" Phương trình  $\Leftrightarrow$  ",  $\text{Tan}\left(5x + \frac{1}{4}\pi\right) = \text{Tan}\left(-x - \frac{1}{6}\pi\right)$

$$\left\{x = -\frac{5}{72}\pi + \frac{1}{6}k\pi\right\}$$

$$k, \varepsilon, Z$$

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d. Dạng 6 hệ thức cơ bản :

$$\sin^2(.) + \cos^2(.) = 1$$

$$\tan(.) \cot(.) = 1$$

Ví dụ :  $\sin\left(2x + \frac{1}{3}\pi\right)^2 + \cos\left(4x + \frac{1}{4}\pi\right)^2 = 1$

> **ptcb6ht(sin,2\*x+Pi/3,cos,4\*x+Pi/4);**

" Cho phương trình : ",  $\sin\left(2x + \frac{1}{3}\pi\right)^2 + \cos\left(4x + \frac{1}{4}\pi\right)^2 = 1$

" Chuyển về  $\Leftrightarrow$  ",  $\sin\left(2x + \frac{1}{3}\pi\right)^2 = 1 - \cos\left(4x + \frac{1}{4}\pi\right)^2$

" Hay : ",  $\sin\left(2x + \frac{1}{3}\pi\right)^2 = \sin\left(4x + \frac{1}{4}\pi\right)^2$

" TRUONG HOP I : ",  $\sin\left(2x + \frac{1}{3}\pi\right) = \sin\left(4x + \frac{1}{4}\pi\right)$

$$\{x = \frac{1}{24}\pi - \frac{1}{2}k2\pi\}$$

$$\{x = \frac{5}{72}\pi + \frac{1}{6}k2\pi\}$$

$k, \varepsilon, Z$

" TRUONG HOP II : ",  $\sin\left(2x + \frac{1}{3}\pi\right) = -\sin\left(4x + \frac{1}{4}\pi\right)$

$$\{x = -\frac{7}{72}\pi + \frac{1}{6}k2\pi\}$$

$$\{x = -\frac{11}{24}\pi - \frac{1}{2}k2\pi\}$$

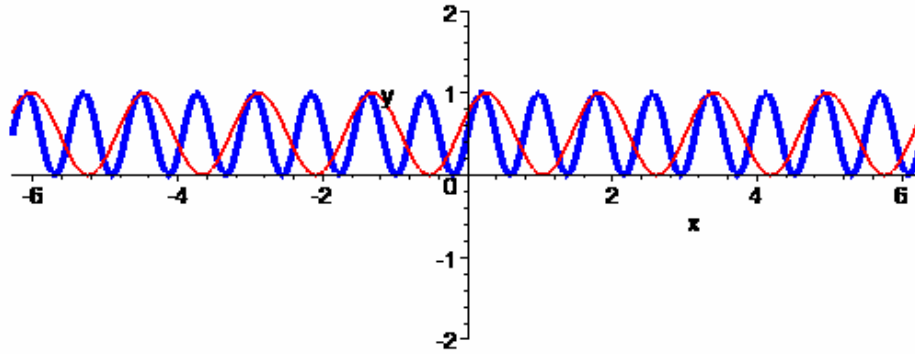
$k, \varepsilon, Z$

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Warning, the name changecoords has been redefined



———      vp-DO  
———      vt-XANH

Ví dụ :  $\tan\left(5x + \frac{1}{3}\pi\right) \cot\left(x + \frac{1}{6}\pi\right) = 1$

> **ptcb6ht(tan, 5\*x+Pi/3, cot, x+Pi/6);**

" Cho phương trình : ",  $\tan\left(5x + \frac{1}{3}\pi\right) \cot\left(x + \frac{1}{6}\pi\right) = 1$

" Biến đổi ",  $1 = \tan\left(5x + \frac{1}{3}\pi\right) \cot\left(5x + \frac{1}{3}\pi\right)$

" Phương trình  $\Leftrightarrow$  ",  $\tan\left(5x + \frac{1}{3}\pi\right) \cot\left(x + \frac{1}{6}\pi\right) = \tan\left(5x + \frac{1}{3}\pi\right) \cot\left(5x + \frac{1}{3}\pi\right)$

" TRUONG HOP I : ",  $\tan\left(5x + \frac{1}{3}\pi\right) = 0$

$$\left\{x = -\frac{1}{15}\pi + \frac{1}{5}k\pi\right\}$$

$k, \varepsilon, Z$

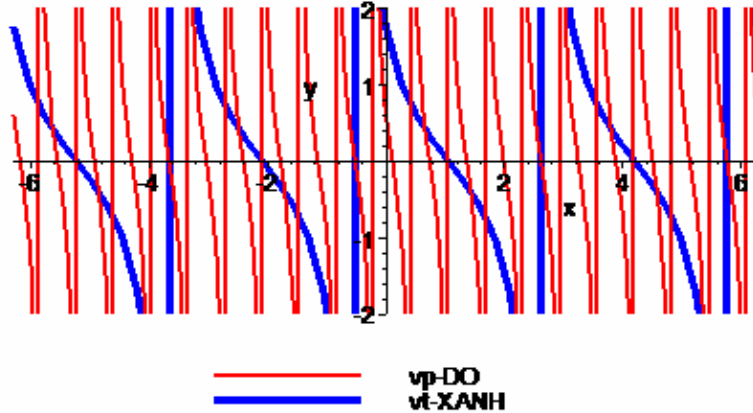
" TRUONG HOP II : ",  $\cot\left(5x + \frac{1}{3}\pi\right) = \cot\left(x + \frac{1}{6}\pi\right)$

$$\left\{x = -\frac{1}{24}\pi + \frac{1}{4}k\pi\right\}$$

$k, \varepsilon, Z$

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e. Dạng áp dụng công thức cộng :

$$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$$

$$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$$

$$\tan(a \pm b) = \frac{\tan a \pm \tan b}{1 \mp \tan a \tan b}$$

Ví dụ : Giải các phương trình sau

$$1. \cos x(\sin 3x + 1) = \cos 3x \sin x$$

$$2. \sin x \sin 2x - \sin 3x = \cos x \cos 2x$$

Giải .

$$1. \cos x(\sin 3x + 1) = \cos 3x \sin x$$

$$\Leftrightarrow \sin 3x \cos x + \cos x = \cos 3x \sin x$$

$$\Leftrightarrow \sin 3x \cos x - \cos 3x \sin x + \cos x = 0$$

$$\Leftrightarrow \sin(3x - x) + \cos x = 0$$

$$\Leftrightarrow \cos x = -\sin 2x = \sin(-2x)$$

$$\Leftrightarrow \cos x = \cos\left(\frac{\pi}{2} + 2x\right)$$

> `ptcb(cos, 2*x+Pi/2, x);`

" Cho phương trình : ",  $-\sin(2x) = \cos(x)$

" Phương trình tương đương "

$$2x + \frac{\pi}{2} = x + k2\pi$$

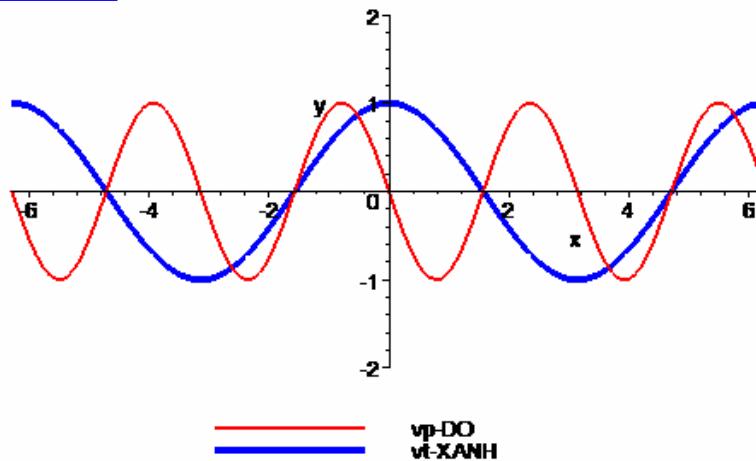
$$2x + \frac{\pi}{2} = -x + k2\pi$$

$$\{x = k2\pi - \frac{1}{2}\pi\}$$

$$\{x = \frac{1}{3}k2\pi - \frac{1}{6}\pi\}$$

$k, \epsilon, Z$

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f. Dạng áp dụng công thức nhân :

Lưu ý :

$$\begin{aligned} \cos 2(.) &= \cos^2(.) - \sin^2(.) \\ &= 2\cos^2(.) - 1 \\ &= 1 - 2\sin^2(.) \\ \sin 2(.) &= 2\sin(.)\cos(.) \end{aligned}$$

ví dụ : giải các phương trình sau

$$1. \frac{\cos 3x}{\sin x} - \frac{\sin 3x}{\cos x} = 2$$

$$2. \frac{\cos 3x}{\cos 2x} - \frac{\sin 3x}{\sin 2x} = 4 \cos 4x$$

Giải .

$$1. \frac{\cos 3x}{\sin x} - \frac{\sin 3x}{\cos x} = 2 \quad (\text{đk : } \sin x \neq 0, \cos x \neq 0)$$

$$pt \Leftrightarrow \cos 3x \cos x - \sin 3x \sin x = 2 \sin x \cos x$$

$$\cos(3x + x) = \sin 2x \Leftrightarrow \cos 4x = \cos(p/2 - 2x)$$

$$2. \frac{\cos 3x}{\cos 2x} - \frac{\sin 3x}{\sin 2x} = 4 \cos 4x \quad (\text{đk : } \sin 2x \neq 0, \cos 2x \neq 0)$$

$$pt \Leftrightarrow \sin 2x \cos 3x - \cos 2x \sin 3x = 4 \sin 2x \cos 2x \cos 4x$$

$$\Leftrightarrow \sin(2x - 3x) = 2 \sin 4x \cos 4x \Leftrightarrow \sin(-x) = \sin 8x$$

g. Dạng áp dụng công thức biến đổi tổng – tích :

$$\cos + \cos = 2 \cos \cos$$

$$\cos - \cos = -2 \sin \sin$$

$$\sin + \sin = 2 \sin \cos$$

$$\sin - \sin = 2 \cos \sin$$

$$a, b \text{ --- } > \left( \frac{a+b}{2} \right) \left( \frac{a-b}{2} \right)$$

ví dụ : giải các phương trình sau

$$1. \sin(3x) + \sin(x) + \sin(2x) = 0$$

> `ptcbctbd(sin, 3*x, x, 2*x);`

" Cho phương trình : ",  $\sin(3x) + \sin(x) + \sin(2x) = 0$

" Áp dụng công thức biến đổi TONG ---> TICH "

$$2 \sin(2x) \cos(x) + \sin(2x) = 0$$

" Phương trình  $\Leftrightarrow$  ",  $(2 \cos(x) + 1) \sin(2x) = 0$

" TRUONG HOP I : ",  $\sin(2x) = 0$

$$\left\{ x = \frac{k2\pi}{2} \right\}$$

$$\left\{ x = \frac{1}{2}\pi + \frac{1}{2}k2\pi \right\}$$

$$k, \epsilon, Z$$

" TRUONG HOP II : ",  $2 \cos(x) + 1 = 0$

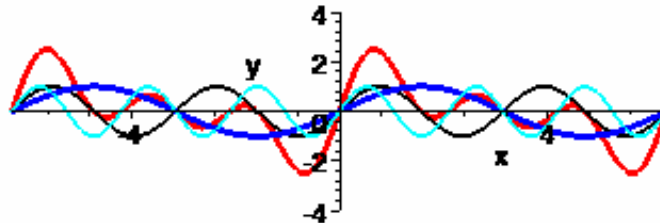
$$\left\{ x = \frac{2}{3}\pi + k2\pi \right\}$$

$$\left\{ x = -\frac{2}{3}\pi + k2\pi \right\}$$

$$k, \epsilon, Z$$

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— Sin U-CYAN  
— Sin V-XANH  
— Sin W-DEN  
— Tong-DO

2.  $\cos(4x) + \cos(3x) + \cos(2x) = 0$

> **ptcbctbd(cos, 4\*x, 2\*x, 3\*x);**

" Cho phương trình : ",  $\cos(4x) + \cos(3x) + \cos(2x) = 0$

" Áp dụng công thức biến đổi TONG ---> TICH "

$$2 \cos(3x) \cos(x) + \cos(3x) = 0$$

" Phương trình  $\Leftrightarrow$  ",  $(2 \cos(x) + 1) \cos(3x) = 0$

" TRUONG HOP I : ",  $\cos(3x) = 0$

$$\left\{ x = \frac{1}{6} \pi + \frac{1}{3} k 2 \pi \right\}$$

$$\left\{ x = -\frac{1}{6} \pi + \frac{1}{3} k 2 \pi \right\}$$

$$k, \varepsilon, Z$$

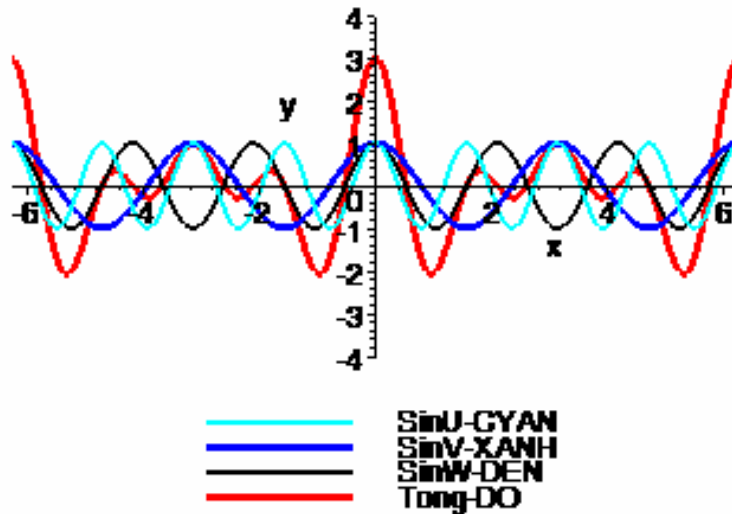
" TRUONG HOP II : " ,  $2 \cos(x) + 1 = 0$

$$\left\{ x = \frac{2}{3} \pi + k 2 \pi \right\}$$

$$\left\{ x = -\frac{2}{3} \pi + k 2 \pi \right\}$$

$$k, \varepsilon, Z$$

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h. Dạng  $A^{2m} + B^{2n} = 0$

$$A^{2m} + B^{2n} = 0 \Leftrightarrow \begin{cases} A = 0 \\ B = 0 \end{cases}$$

Ví dụ :

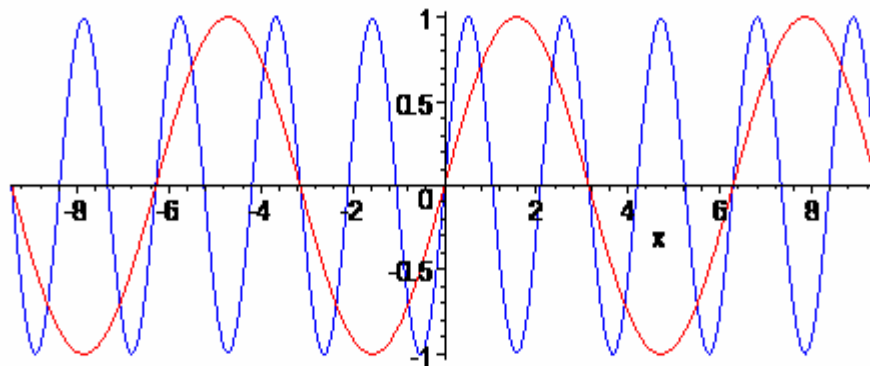
$$\sin^2 x + \sin^2 3x = 0$$

$$pt \Leftrightarrow \begin{cases} \sin x = 0 \\ \sin 3x = 0 \end{cases} \Leftrightarrow \begin{cases} x = kp \\ 3x = hp \end{cases} \quad (k, h \in \mathbb{Z})$$

$$\Leftrightarrow kp = \frac{hp}{3} \Leftrightarrow h = 3k \quad (k \in \mathbb{Z})$$

```
> solve({sin(x+k*2*Pi)=0,sin(3*x+h*2*Pi)=0},{x,k,h});
      {h=3k,k=k,x=-2k*pi}
```

```
> plot([sin(x),sin(3*x)],x=-3*Pi..3*Pi,color=[red,
blue],legend=['red_sinx','blue_sin3x']);
```



— red\_sinx  
— blue\_sin3x